International Conference on Control, Engineering & Information Technology (CEIT'13)
Proceedings Engineering & Technology - Vol.3, pp. 51-56, 2013

Decoupled State Feedback Control design for Two-Wheeled Mobile Robot: An LMI approach

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***Abstract*— This paper deals with the quadratic stabilization problem of nonlinear systems represented by a decoupled multi-model. Based on a linearization based-decoupled multi-model structure, the state feedback controller which guarantees the stability of the closed loop multi-model is designed using a common quadratic Lyapunov function CQLF in terms of LMIs constraints. To improve the stabilization performances, a relaxation of CQLF is presented. The proposed control structure is illustrated to the stabilization of Two-Wheeled Inverted Pendulum Mobile Robot and the simulation results showed the good performances of proposed stabilization approach.**

***Keywords*— decoupled multi-model, state feedback control, CQLF, relaxed CQLF, LMIs, Two-Wheeled Inverted Pendulum Mobile Robot.**

I. Introduction

In the recent years, there is a growing interest to the multi-model approach for the representation and control of nonlinear complex systems. The multi-model approach proved that it is a powerful and efficient tool for modelling and control of this class of systems thanks to its capability to take into account the presence of multiple operating regimes of the system [10]. The main idea of the multi-model is the decomposition of the nonlinear system behaviour into a finite number of operating zones; each is represented by a local linear sub-model. Subsequently, the global multi-model output is carried out by the aggregation of sub-models. According to the way that the sub-models are combined, two well-known multi-models are proposed in the literature: coupled states multi-model (or Takagi-Sugeno model) and decoupled states multi-model (or decoupled multi-model) [4][5][10][11][12]. The coupled states multi-model or the conventional Takagi-Sugeno model is the most used in the most research works. However, studies on decoupled multi-model remains restricted compared to the first type of multi-model. In the other hand, finding a suitable decomposition technique for the system to obtain a family of sub-models is the first step to multi-model design. Several techniques are proposed in the literature whose three methods namely the convex polytopic transformation method, the identification technique and the linearization method, are the frequently used. The last one is based on the linearization of the nonlinear system around a finite number of operating points and then obtained a family of local linear sub-models. Forasmuch that the convex polytopic transformation method is conventionally used, we chose to use the linearization technique in our study. However, the drawback of this approach is the determination of the relevant operating points. Our idea is to exploit a fuzzy clustering algorithm, like as FCM (fuzzy c-means)[13], to find the cluster centers and then consider these centers as the desired operating points. Once the linearization-based decoupled multi-model is obtained, the next step is to design a decoupled static state feedback controller which stabilizes the closed-loop system. Sufficient stability conditions are presented in terms of Linear Matrix Inequalities constraints which guarantee the quadratic stability of the decoupled multi-model by using common quadratic Lyapunov functions (CQLF). In [2][9], relaxations of stability conditions are proposed to improve the stabilization performances of the Takagi-Sugeno model that offer more reliability and effectiveness of the stabilizing controller and the desired performances.

In our work, we try to exploit these relaxations to stabilize quadratically a highly nonlinear system by a decoupled static state feedback control. The paper is organized as follow: In section II, the Two Wheeled Inverted Pendulum Mobile Robot is presented. Section III is devoted to the decoupled multi-model design which contains the linearization technique principle based on the FCM to determine the operating points, as well as the global structure of the decoupled multi-model. Decoupled control, basic and relaxed stabilization conditions are presented in section IV. In section V, some simulation results for the decoupled control of the Two Wheeled mobile Robot are given, and finally, concluding remarks are located in the last section.

II. The Two Wheeled Inverted Pendulum Mobile Robot

A. Description

The Two Wheeled Inverted Pendulum Mobile Robot is widely used and applied for the validation of various control strategies. This intelligent robot is based on the inverted pendulum principle. Its mechanical structure and characteristics are very complex and it is considered as a highly nonlinear complex system. The Two-wheeled inverted pendulum robot has just two driving wheels for actuator. However, it has three degrees of freedom; two of planar motions and one of tilt-angular motion. In the literature, several studies have been carried out for the analysis [17] and the stabilization [1] of the two-wheeled inverted pendulum robot. Figure 1 shows the principle and the coordinate system of the Robot.

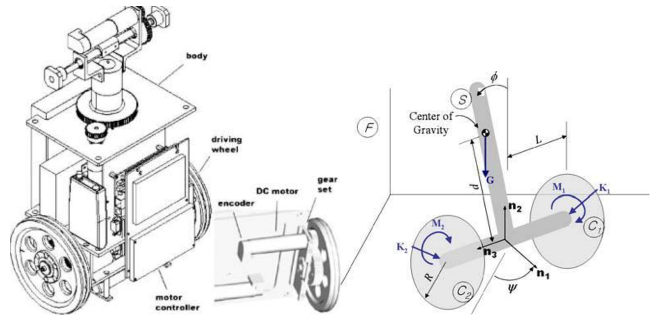

Fig. 1 The Two Wheeled Inverted Pendulum Mobile Robot

B. Dynamic Model

Three motion equations could describe the plant dynamics [18]:

$$3(m_s+m_c)\ddot{p}-m_s d\cos\varphi\ddot{\varphi}+m_s d\sin\varphi(\dot{\varphi}+\dot{\psi})=-\left(\frac{u_1+u_2}{R}\right) \quad (1)$$

$$\left[\left(3L^2+\frac{1}{2R^2}\right)m_c+m_s d^2\sin^2\varphi+I_2\right]\ddot{\psi}+m_s d^2\sin\varphi\dot{\varphi}\dot{\psi}=\frac{L(u_1-u_2)}{R} \quad (2)$$

$$m_s d\left(\cos\varphi\ddot{p}+d\sin\varphi\cos\varphi\dot{\varphi}^2+g\sin\varphi\right)-\left(m_s d^2+I_3\right)\ddot{\varphi}=u_1+u_2 \quad (3)$$

Where u_1 and u_2 are the torques applied to the robot wheels.

Let's consider $[p,\dot{p},\varphi,\dot{\varphi},\psi,\dot{\psi}]^T$ as the state vector of the robot. Then, a mathematical development leads to the following state space representation:

$$\begin{bmatrix}\dot{p}\\ \ddot{p}\\ \dot{\varphi}\\ \ddot{\varphi}\\ \dot{\psi}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&\alpha_{24}&0&\alpha_{26}\\0&0&0&1&0&0\\0&0&0&\alpha_{44}&0&\alpha_{46}\\0&0&0&0&0&1\\0&0&0&0&0&\alpha_{66}\end{bmatrix}\begin{bmatrix}p\\ \dot{p}\\ \varphi\\ \dot{\varphi}\\ \psi\\ \dot{\psi}\end{bmatrix}+\begin{bmatrix}0&0\\ \beta_{21}&\beta_{22}\\0&0\\ \beta_{41}&\beta_{42}\\0&0\\ \beta_{61}&\beta_{62}\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}+\begin{bmatrix}0\\h_2\\0\\h_4\\0\\0\end{bmatrix} \quad (4)$$

With

$$\alpha_{24}=\frac{m_s d\cos\varphi\left(m_s d^2+I_3\right)\tan\varphi\dot{\psi}}{m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$\alpha_{26}=\frac{m_s d\cos\varphi\left(m_s d^2+I_3\right)\tan\varphi\dot{\varphi}}{m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$\alpha_{44}=\frac{m_s d\sin\varphi\dot{\psi}}{m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$\alpha_{46}=\frac{-m_s{}^2d^2\cos\varphi\sin\varphi\left(2m_s+3m_c\right)\dot{\varphi}}{m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$\alpha_{66}=\frac{-m_s d^2\sin\varphi\cos\varphi\dot{\varphi}}{m_c\left(3L^2+\frac{1}{2R^2}\right)+m_s{}^2d^2\sin^2\varphi+I_2}$$

$$\beta_{21}=\beta_{22}=\frac{m_s d\left(R\cos\varphi+d\right)+I_3}{R\left(m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)\right)}$$

$$\beta_{41}=\beta_{42}=\frac{3\left(m_c+m_s\right)}{m_s{}^2d^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$\beta_{61}=-\beta_{62}=\frac{L/R}{m_c\left[3L^2+\frac{1}{2R^2}\right]+m_s{}^2d^2\sin^2\varphi+I_2}$$

$$h_2=\frac{-m_s{}^2gd^2\sin\varphi\cos\varphi}{m_s{}^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

$$h_4=\frac{-3\left(m_c+m_s\right)m_s gd\sin\varphi}{m_s{}^2\cos^2\varphi-3\left(m_s d^2+I_3\right)\left(m_c+m_s\right)}$$

Table 1 shows the parameters of the robot.

TABLE I
PARAMETERS OF THE PLANT

Parameter	Designations	Values
m_s	*Mass of the body (kg)*	*4.315*
m_c	*Mass of a wheel (kg)*	*0.503*
I_2	*n2- directional rotational inertia of the body (kg.m²)*	*0.003679*
I_3	*n3- directional rotational inertia of the body (kg.m²)*	*0.02807*
R	*Radius of a wheel (m)*	*0.073*
L	*Half-distance between wheels*	*0.1*
d	*Distance from center to gravity center (m).*	*0.1*

III. DECOUPLED MULTI-MODEL

Obtaining a decoupled multiple model structure from a nonlinear system has 3 steps:

- The selection of a technique for the decomposition of the nonlinear system operating in local areas.
- The design of a family of local linear models representing the functioning of zones already selected.

- The aggregation of sub models using weighting functions defining the contribution of each sub-model at any time in the system.

A. Linearization-based multi-model

The first step for designing a multiple model is to choose a decomposition technique of the nonlinear system behaviour. In our work, we choose to utilize the Linearization method which is described as follow:
Let's consider a nonlinear system represented by (5):

$$\begin{cases} \dot{x}(t) = f\left(x,u,t\right) \\ y(t) = g\left(x,t\right) \end{cases} \tag{5}$$

Where $x \in \Re^n, u \in \Re^m, y \in \Re^p$ represent, respectively the state, the input and the output vectors of the plant.

The Linearization of the system (5), using the Taylor series method, around an arbitrary operating point $\left(x_{oi}, u_{oi}\right)$ is given by:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + R_i \\ y_i(t) = C_i x_i(t) \end{cases} \tag{6}$$

With

$$A_i = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x_{oi}\\u=u_{oi}}}, B_i = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x_{oi}\\u=u_{oi}}}$$

$$R_i = f\left(x_{oi}, u_{oi}\right) - A_i x_{oi} - B_i u_{oi}$$

The question now posed is how to choose or determine the relevant operating points?

B. FCM based operating points computing:

As mentioned in section I, our idea is to use a fuzzy clustering algorithm, for example the FCM (fuzzy c-means) algorithm (see [13] for more details). This algorithm allows decomposing the nonlinear system operation into a set of operating zones (clusters), and returns the center coordinates as well as a fuzzy partition matrix U which determines the fuzzy membership degree of each cluster (sub-model) in each instant.
In our work, we used the MATLAB Toolbox function defined as follow:

$$\left[center, U, obj\_fcn\right] = fcm\left(data, cluster\_n\right)$$

The last step of the decoupled multi-model design is the aggregation of the sub-models. A decoupled multi-model takes its name from the combination of sub-models in the form of a decoupled state structure (see figure 2) [11][12]. Based on system (2), a decoupled multi-model based on Linearization is given by (7):

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + R_i \\ y_i(t) = C_i x(t) \end{cases} \tag{7}$$

$$y_{MM}(t) = \sum_{i=1}^{r} \mu_i\left(\xi(t)\right) y_i(t) \tag{8}$$

Where $\mu_i(\xi)$ is the weighted function of the i^{th} sub-model, defined by (7+8) and r is the number of local sub-models.

$$\mu_i\left(\xi(t)\right) = \frac{w_i\left(\xi(t)\right)}{\sum_{j=1}^{r} w_j(\xi(t))} \tag{9}$$

With

$$w_i\left(\xi(t)\right) = \exp\left(-\frac{1}{2}\frac{\left[\xi(t) - \varsigma_i\right]^2}{\sigma^2}\right) \tag{10}$$

$\xi(t)$ is called decision variable, which can be the input, the output, a state , … of the system. ς_i is the cluster center or the operating point and σ is the dispersion of the Gaussian function (9).
A multi-model with decoupled states is given by figure 2.

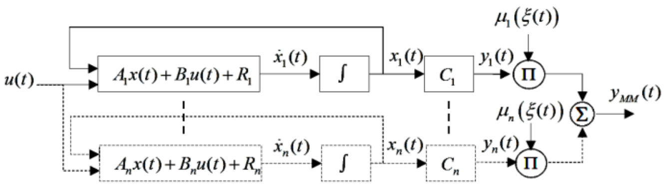

Fig. 2 Linearization-based decoupled multi-model architecture

IV. Decoupled quadratic stabilization by a Static State feedback

In this section, the goal is to establish a decoupled state feedback control law which stabilizes quadratically the decoupled multi-model and likewise the nonlinear system in the closed Loop.

A. State feedback controller

For each sub-model, a corresponding local controller given by (11), which stabilizes it in closed loop, is designed:

$$u_i(t) = -F_i x(t) \tag{11}$$

Then, the global control law is obtained by the weighted sum of local controller as follow:

$$u(t) = -\sum_{i=1}^{r} \mu_i\left(\xi(t)\right) u_i(t) = -\sum_{i=1}^{r} \mu_i\left(\xi(t)\right) F_i x(t) \tag{12}$$

The local gain matrix F_i (*for i=1,…,r*) is determined by LMIs techniques such that some stability conditions are satisfied.

B. Quadratic stabilization using CQLF

In this section, the gain matrix F_i is synthesized based on LMIs and using a common quadratic Lyapunov function for stability conditions.

Theorem IV.1 The decoupled multi-model (7+8) is quadratically stabilizable via the state feedback control (11), if there exist a common positive definite matrix $P > 0$ and matrices K_i , *for i=1,...,r,* such that the following LMIs conditions are satisfied:

$$\begin{cases} \Xi(G_{ii}) < 0, & i = j \\ \Xi(G_{ij}) \le 0 & i < j \le r \end{cases} \quad (13)$$

With $\Xi(G_{ij}) = \left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right)$ (14)

And $G_{ij} = A_i + B_i F_j$ (15)

The mathematical development leads to the following LMIs stability conditions:

The closed Loop decoupled multi-model is quadratically stabilizable if there exist a common definite positive matrix $X = P^{-1} > 0$, and matrix K_i such that:

$$X > 0 \quad (16.a)$$

$$A_i X + X A_i^T - B_i K_i - K_i^T B_i^T < 0 \quad \forall\, i = j \quad (16.b)$$

$$A_i X + A_j X + X A_i^T + X A_j^T - B_i K_j - B_j K_i - K_j^T B_i^T - K_i^T B_j^T \le 0 \;\; \forall\, i < j \le r \quad (16.c)$$

Then $F_i = K_i X^{-1}$ and $P = X^{-1}$

C. Quadratic stabilization using relaxed LMIs conditions

To further reduce the degree of conservatism of the results above, a relaxation LMIs conditions proposed in [2] in the following theorem:

Theorem IV.2 If there exist a symmetric matrix $P > 0$, matrices Q_{ij} and matrices N_i , $\forall\, i = 1,...,r$ which satisfy:

$$\begin{cases} \Xi(G_{ii}) < Q_{ii}, & i = j \\ \Xi(G_{ij}) \le Q_{ij} & i < j \le r \end{cases} \quad (17)$$

$$\begin{pmatrix} Q_{11} & \cdots & Q_{1r} \\ \vdots & \ddots & \vdots \\ Q_{r1} & \cdots & Q_{rr} \end{pmatrix} < 0 \quad (18)$$

Or also:

$$X > 0 \quad (19.a)$$

$$A_i X + X A_i^T - B_i N_i - N_i^T B_i^T < Q_{ii} \quad \forall\, i = j \quad (19.b)$$

$$A_i X + A_j X + X A_i^T + X A_j^T - B_i N_j - B_j N_i - N_j^T B_i^T - N_i^T B_j^T \le Q_{ij} \quad \forall\, i < j \le r \quad (19.c)$$

And $\begin{cases} Q_{ii} < 0, & i = j \\ Q_{ij} < 0, & i < j \le r \end{cases}$ (19.d)

Then the decoupled multi-model (7) is quadratically stabilizable via the decoupled state feedback controller (11).

And we have $F_i = N_i X^{-1}$

V. SIMULATIONS

In this section, a decoupled static state feedback control is designed to stabilize the Two Wheeled Inverted Pendulum Mobile robot described in section II.

As mentioned previously, the decoupled multi-model is designed using the linearization technique and the FCM algorithm to compute the centers of clusters which are the desired operating points. Five operating points (x_{oi}, u_{oi}) are selected.

$$x_{oi} = \begin{bmatrix} x_{oi1} \\ x_{oi2} \\ x_{oi3} \\ x_{oi4} \\ x_{oi5} \\ x_{oi6} \end{bmatrix} = \begin{bmatrix} -1.8929 & -3.5429 & -3.4844 & -1.9969 & -1.2013 \\ -0.6635 & -0.3462 & -0.5700 & -0.4070 & -0.1684 \\ -0.0204 & -0.0206 & -0.0040 & 0.0050 & -0.4128 \\ -7.8659 & 12.2470 & -12.0374 & 8.6415 & -0.3227 \\ 0.0467 & 0.0400 & 0.0402 & 0.0462 & 0.0494 \\ -0.0016 & -0.0017 & -0.0018 & -0.0016 & -0.0019 \end{bmatrix}$$

$$u_{oi1} = u_{oi2} = \begin{bmatrix} -0.0020 & -0.0115 & 0.0351 & -0.1326 & 0.1905 \end{bmatrix}$$

A. Quadratic State feedback control of robot

Initial conditions: $x_0 = \begin{bmatrix} -0.5 & 0 & \frac{\pi}{20} & 0 & \frac{\pi}{60} & 0 \end{bmatrix}$;

$$P = \begin{bmatrix} 0.0135 & 0.0860 & 0.0371 & 0.0026 & -0.0000 & -0.0002 \\ 0.0860 & 0.6228 & 0.2650 & 0.0205 & -0.0001 & -0.0015 \\ 0.0371 & 0.2650 & 0.1195 & 0.0099 & 0.0000 & -0.0006 \\ 0.0026 & 0.0205 & 0.0099 & 0.0035 & -0.0001 & -0.0002 \\ -0.0000 & -0.0001 & 0.0000 & -0.0001 & 0.0139 & 0.0088 \\ -0.0002 & -0.0015 & -0.0006 & -0.0002 & 0.0088 & 0.0139 \end{bmatrix}$$

The gain matrices F_i are given by:

$$F_1 = \begin{bmatrix} -1.9988 & -15.1320 & -6.5533 & -1.6257 & 20.9986 & 19.4831 \\ 1.5799 & 12.1369 & 5.3195 & 1.4992 & -21.0096 & -19.4921 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 2.0924 & 15.1732 & 6.9922 & 0.8383 & 20.8567 & 19.1936 \\ -2.505 & -18.122 & -8.1598 & -0.9632 & -20.8521 & -19.1781 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -0.726 & -5.3438 & -2.3528 & -0.3627 & 0.0271 & 0.0396 \\ 0.7257 & 5.3413 & 2.3517 & 0.3627 & -0.029 & -0.0427 \end{bmatrix} \times 10^3$$

$$F_4 = \begin{bmatrix} -0.6370 & -4.6884 & -2.0642 & -0.3182 & 0.0261 & 0.0367 \\ 0.6366 & 4.6858 & 2.0631 & 0.3182 & -0.0278 & -0.0395 \end{bmatrix} \times 10^3$$

$$F_5 = \begin{bmatrix} 1.2400 & 9.1262 & 4.0185 & 0.6190 & -0.0088 & -0.0334 \\ -1.2401 & -9.1270 & -4.0189 & -0.6191 & 0.0122 & 0.0387 \end{bmatrix} \times 10^4$$

Fig 3, 4 and 5 represent, respectively, the quadratic stabilization via the decoupled static state feedback control of the position, angle ψ and the angle φ outputs of the two wheeled robot using CQLF and relaxed CQLF.

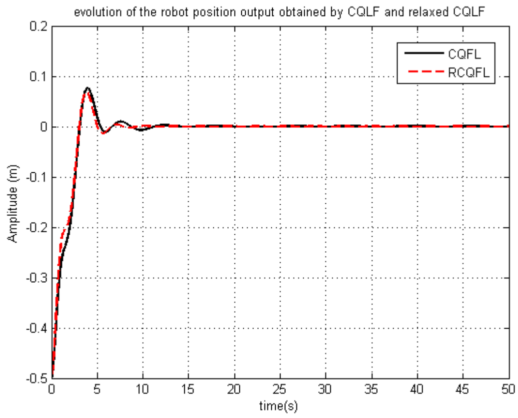

Fig. 3 Quadratic stabilization of the position output of the robot using CQLF compared to RQLF

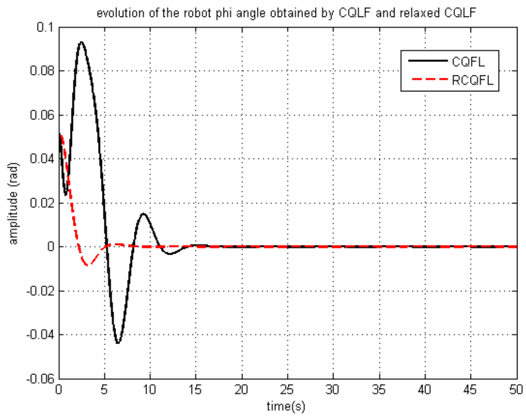

Fig. 4 Quadratic stabilization of the ψ output of the robot using CQLF compared to RQLF.

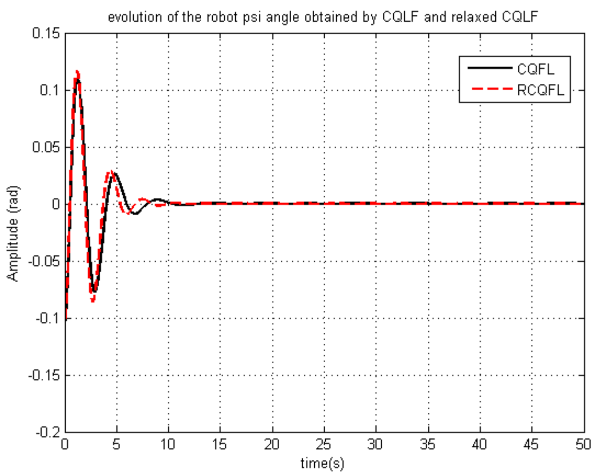

Fig. 5 Quadratic stabilization of the φ output of the robot using CQLF compared to RQLF.

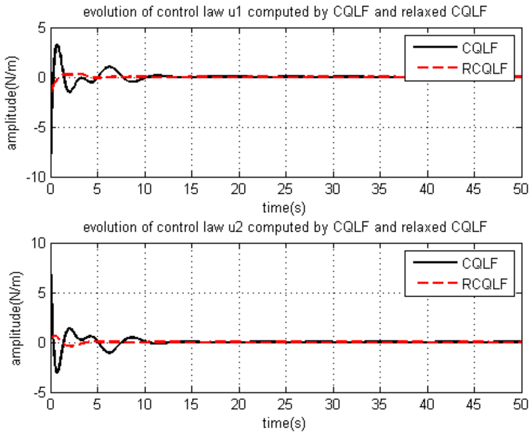

Fig. 6 Evolution of the control laws applied to system.

Through figures 3, 4 and 5, we can see satisfying performances of the decoupled state feedback control of the system and the improvement obtained by adding the relaxation on stability conditions.

Figure 6 shows the control laws applied to system. We observe clearly the good improvement on the control performances.

VI. Conclusions

This paper presents a decoupled multi-model designed by the Linearization technique and a fuzzy c-means algorithm. A decoupled state feedback control is then synthesized for the quadratic stabilization of two wheeled inverted pendulum mobile robot based on common quadratic Lyapunov function and LMI formulations. Then, to improve the stabilization performances, a relaxation in the stability conditions is presented. The simulation results show a good improvement on the stabilization performances and the stabilizing control laws.

References

[1] A. P. Aguiar, A. N. Atassi and A. Pascoal, "Stabilization of a Nonholonomic dynamic Wheeled Mobile Robot with parametric modeling uncertainty using Lyapunov functions", Proc. of CONTROLO'2000 - 4th Portuguese Conference on Automatic Control, Guimares, Portugal, October 2000.

[2] C.H. Fang, Y.S. Liu, S.W. Kau, L. Hong, and C.H. Lee, "A New LMI-Based Approach to Relaxed Quadratic Stabilization of T–S Fuzzy Control Systems", IEEE Transactions on Fuzzy Systems, vol. 14, No. 3, June 2006.

[3] G. Feng, "A Survey on Analysis and Design of Model-Based Fuzzy Control Systems", IEEE Transactions on Fuzzy Systems, vol. 14, issue 5, pp. 676- 697, 2006.

[4] G. Z. Angelis, "System Analysis, Modelling and Control with Polytopic Linear Models", Technische Universiteit Eindhoven, 2001.

[5] Chadli M., Maquin, D. Ragot J., "Static Output Feedback for Takagi-Sugeno Systems: An LMI Approach", Proceedings of the 10th Mediterranean Conference on Control and Automation - MED2002, Lisbon, Portugal, July 9-12, 2002.

[6] Chadli M., Ragot J., Maquin D., "Quadratic stability and stabilisation of interval Takagi-Sugeno model: LMI approach", Proceedings of the 2005 IEEE International Symposium on Intelligent Control Limassol, Cyprus, Page(s): 1035- 1038, June 27-29, 2005.

[7] S.W. Nawawi, M.N Ahmad, J.H.S Osman, "Control of Two-wheels Inverted Pendulum Mobile Robot Using Full Order Sliding Mode Control", Proceedings of International Conference on Man-Machine Systems 2006, September 15-16, 2006, Langkawi, Malaysia.

[8] N. M. Abdul Ghani, N. I. Mat Yatim, and N. A. Azmi, "Trajectory Tracking Of Two Wheels Mobile Robot Using Sliding Mode Control", Proceedings of the fourth Global Conference on Power Control and Optimization, vol. 1337, pp. 135-139, 2–4 December 2010, Sarawak, Malaysia.

[9] Korba P., Babuska R., Verbruggen H. B., and M. Frank P., "Fuzzy Gain Scheduling: Controller and Observer Design Based on Lyapunov Method and Convex Optimization", IEEE Transactions on Fuzzy Systems, vol. 11, No. 3, June 2003.

[10] R. Murray-Smith, T. A. Johansen, "Multiple Model Approaches to Nonlinear Modelling and Control", Taylor and Francis, 1997.

[11] R. Orjuela, D. Maquin and J. Ragot, "Non-Linear System Identification Using Uncoupled State Multiple-Model Approach", Workshop on Advanced Control and Diagnosis, ACD'2006, Nancy, France, 2006.

[12] R. Orjuela, B. Marx, J. Ragot, D. Maquin, "State estimation for non-linear systems using a decoupled multiple model", International Journal of Modelling, Identification and Control, vol. 4, no. 1, 2008.

[13] R. Babuska. "Fuzzy Modeling for Control", Kluwer Academic Publishers, Boston, 1998.

[14] S. Boyd, V. Balakrishnan, E. Feron, and L. El Ghaoui, "Control system analysis and synthesis via linear matrix inequalities", American Control Conference, 2:2147-2154, San Francisco, June 1993.

[15] S. Petterson, B. Lennartson, "An LMI Approach for Stability Analysis of Nonlinear Systems", Proc. of European Control Conference, ECC'97, 1-4 July 1997, Brussels, Belgium.

[16] S. H. Jeong and Takayuki Takahashi, "Wheeled Inverted Pendulum Type Assistant Robot: Inverted Mobile, Standing, and Sitting Motions", Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, Page(s): 1932-1937, San Diego, CA, USA, Oct 29 - Nov 2, 2007.

[17] Wang, H.O., Tanaka, K. and Griffin, M.F. "An approach to fuzzy control of nonlinear systems: stability and design issues", IEEE Transactions on Fuzzy Systems, vol. 4, issue 1, pages 14-23, Feb 1996.

[18] Y. KIM, S. H. KIM and Y. K. KWAK, "Dynamic Analysis of a Nonholonomic Two-Wheeled Inverted Pendulum Robot", Journal of intelligent and Robotic Systems, vol. 44, pp. 25-46, 2005.